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Exact numerical reasoning in blind children and adults

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ABSTRACT

What is the origin of exact numerical reasoning in humans? Previous studies report that innumerate humans are unable to recognize that two sets placed in one-to-one correspondence are exactly equal, and argue that this ability emerges when humans acquire symbolic numbers. Here we tested blind children and adults ($n = 140$), and found that despite having symbols to label and count large quantities, these participants did not use one-to-one correspondence to match large sets. We also found that blind individuals who had a gradient of visual sensitivity were more likely to match sets exactly, but participants who only used haptic exploration did not. Thus, numerate individuals (who can sometimes match sets exactly) are nevertheless unable to express their understanding of exact equality in a commonly used measure of this ability. We conclude that expressing knowledge of exact equality via set-matching requires not only conceptual understanding, but also experience with sophisticated sensori-motor procedures, suggesting a possible re-analysis of studies with innumerate and semi-numerate humans.

Humans are unique in our capacity to reason about exact number. In most industrialized societies, numerate adults express number through external symbols including verbal number, body counts, written numerals, and physical calculators (Bender & Beller, 2012; Chrisomalis, 2020; Ifrah, 2000; Núñez, 2017; Saxe, 1981; Schmandt-Besserat, 1992). Prominent accounts of numerical cognition argue that a concept of exact number depends on learning symbolic number systems, including number words and counting. In the absence of symbols, innumerate adults, young children, and non-human animals appear to lack exact number concepts, and are limited to noisy and error-prone approximate representations (Carey & Barner, 2019; Dehaene, 1997; Everett, 2013; Feigenson, Dehaene and Spelke, 2004; Gelman & Butterworth, 2005; Gordon, 2004; Laurence & Margolis, 2005; Pica, Lemer, Izard, & Dehaene, 2004; Pitt, Gibson, & Piantadosi, 2022; Rips, Bloomfield, & Asmuth, 2008; Spelke, 2003). However, significant controversy surrounds the evidence regarding such claims, and whether tasks that find differences in exact number concepts are valid tests of exact number knowledge, or might instead also draw on additional knowledge of culturally determined practices and procedures (Frank, Fedorenko, Lai, Saxe, & Gibson, 2012; Laurence & Margolis, 2007; c.f. Everett, 2013).

Evidence for the idea that exact number knowledge depends on symbolic representations comes from studies of innumerate groups like the Pirahã, an indigenous Amazonian group which has no exact number words, counting routines, or written numerals (Gordon, 2004; Everett & Madora, 2012; see also Pica et al., 2004). In one study of the Pirahã, Gordon (2004) evaluated non-verbal knowledge of exact number by testing adults' understanding of Hume's principle — i.e., that two sets placed in one-to-one correspondence are exactly equal (Boolos, 1986; Hume, 1793). He reasoned that if exact number knowledge can emerge independent of symbolic forms, then participants should succeed in matching large sets despite not knowing any number words, or how to count. In contrast, if knowledge of this principle depends upon number words and counting, then the Pirahã should fail to match sets exactly. To test this, Gordon presented participants with a row of objects (e.g., 8 AA batteries) and asked them to match this array with another set of objects (Fig. 1). To succeed on the task, participants were required to align each item in the experimenter's set with an item from their own set, resulting in a one-to-one correspondence between them. Consistent with a role for language, most Pirahã participants in the study made exact matches for small sets up to 3 or 4, but not larger sets. Crucially, this contrasts with results from numerate adults in the US, who readily make exact matches for large sets even when prevented from counting by a verbal shadowing task (Frank et al., 2012). Based on these findings, Gordon concluded that the ability to represent large exact number depends on symbolic forms, such as number words or written numerals.

While Gordon's study concluded that exact number concepts depend on language, other studies have found conflicting evidence regarding this claim (Butterworth & Reeve, 2008; Flaherty & Senghas, 2011;

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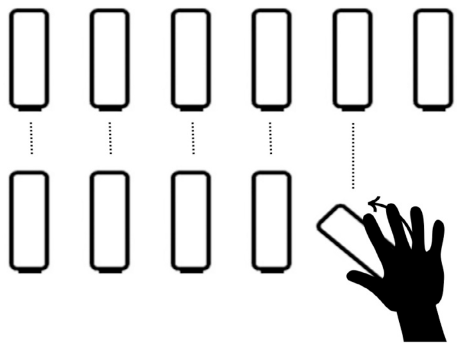

Fig. 1. Example of the task set-up for the parallel set-matching task used in Gordon (2004).

Frank, Everett, Fedorenko, & Gibson, 2008; Jara-Ettinger, Piantadosi, Spelke, Levy, & Gibson, 2017; Pica et al., 2004; Schneider, Brockbank, Feiman, & Barner, 2022). For example, in a study that attempted to replicate Gordon (2004), Frank et al. (2008) tested a different group of Pirahã adults in the set-matching task, and found that they produced exact matches for large sets when given feedback, suggesting that participants in Gordon's original study may not have understood the task, and may have had exact number concepts. However, challenging this conclusion, Everett and Madora (2012) argued that participants in Frank et al.'s study likely received explicit training on both one-to-one correspondence and exact number words, and reported new evidence replicating Gordon's findings. Meanwhile, studies with other innumerate and semi-numerate groups find mixed results (Butterworth & Reeve, 2008; Flaherty & Senghas, 2011; Jara-Ettinger et al., 2017; Pica et al., 2004). As one example of this, in a study of the Tsimané – a farming-foraging community whose children master counting at a delayed age relative to children in other cultures – Jara-Ettinger et al. (2017) found that some participants made exact matches before learning how to count, while others did not, even after learning to count. Taken together, past studies raise questions about the link between learning a verbal number system and understanding Hume's principle.

In an effort to address these conflicting findings, a recent study of US preschoolers examined how changes in early numeracy relate to children's ability to exactly match large sets in the set-matching task (Schneider et al., 2022). Schneider et al. reasoned that if the ability to match large sets depends on learning number words and counting routines, then only children who have acquired a system of counting should succeed at matching large sets. To probe this, they tested 3- to 5-year-old children with variable counting skills on an adaptation of Gordon's (2004) set-matching task, wherein children were asked to match sets of fish without counting. Consistent with the idea that learning to count might cause a conceptual change, Schneider et al. found that children could not exactly match large sets until after they had learned to accurately count and construct large sets (i.e., children classified as "cardinal principle knowers", as per Wynn, 1990, 1992), even though explicit counting was not allowed during the task.[1] Further, and relevant to the current study, they also found the unexpected result that many children who could count large sets nevertheless failed to exactly match large sets, suggesting that simply learning a verbal counting system did not guarantee the ability to make exact matches, and that additional learning may be required. Schneider et al. reasoned that children who had learned to count might have a concept of exact number – as evidenced by their understanding of some number words and ability to count large sets – but that matching large sets also involves procedural knowledge, and in particular, requires strategies for sequentially matching items in one set to items in another.

On the procedural analogy hypothesis proposed by Schneider et al. (2022), matching large sets requires not only a conceptual understanding of Hume's principle, but also a sensori-motor procedure for organizing items in space, and in particular, for aligning items in one set with items in a second set, making sure that each object is involved in only one pairing. They note that such a "partition procedure" is explicitly present in counting routines (Gelman & Gallistel, 1978; Gelman, 1982), in which children learn to assign each object in a set to only one number word, making sure to set aside counted objects, resulting in a one-to-one correspondence between words and objects, and an accurate cardinal label of the set. Given this, individuals in numerate groups might learn to set-match by first learning a one-to-one procedure for counting, and then analogically extending this procedural knowledge to the problem of set-matching. Specifically, they might learn that, just as objects in a counted set are partitioned when numerical labels are assigned to them, objects in a matched set are also excluded from future consideration once they have been assigned a corresponding object (Fig. 2). Crucially, on this procedural analogy hypothesis, cross-cultural differences in set-matching might therefore arise in one of two ways. First, it's possible that counting does produce a conceptual change, as argued by Gordon (2004) but that in addition it provides a procedural template for organizing objects in space, and for implementing exact matches of large sets. Second, it's possible that young children and innumerate Pirahã participants understand Hume's principle of one-to-one correspondence, but simply lack a procedure for organizing items in space and partitioning matched items from unmatched items. Learning to count might provide a basis for creating this procedure, but not a concept of exact number.

Although the procedural analogy hypothesis might explain the correlation between counting knowledge and one-to-one set-matching, alternative explanations for this relation also exist. For example, some previous studies speculate that data from groups like the Pirahã may underestimate exact number knowledge, either because participants didn't understand that their goal was to make exact matches, or because they found it easier to match using other strategies (Frank et al., 2008; Laurence & Margolis, 2007). Another explanation is that some participants, including young children who are still learning to count, like in Schneider et al.'s study, might prefer to use non-exact strategies like approximation or matching the length of lines when matching sets. One reason for this could be that these strategies can be implemented quickly, by glancing at a target set, and then grabbing a roughly equal set of items to match it (Gelman, 1969; Frydman and Bryant, 1988; Mix, 1999; Piaget, 1965, 1968; Russac, 1978; see Schneider et al., 2022, for discussion). In contrast, a one-to-one strategy cannot be implemented in a single shot, and requires attending to each individual object, making sure to match each one in sequence, and to carefully partition matched items from unmatched items. Thus, exact matching not only requires knowledge of a procedure, but also the will to deploy it instead of a quicker and easier approximation strategy. Given this, it is possible that some participants in past studies understood Hume's principle but used alternative strategies because they were quicker and easier to deploy.

Here we explored the hypothesis that one-to-one set-matching emerges from an analogy to counting procedures by investigating a novel group of participants: blind children and adults. Currently, relatively little is known about numerical cognition in blind individuals. While past studies report that blind individuals often experience delays in learning mathematics (e.g., Klingenberg, Holkesvik, & Augestad, 2019), few studies have tested numerical reasoning in blind adults (e.g., Castronovo & Seron, 2007a–c; Castronovo & Delvenne, 2013; Crollen et al., 2014; Dormal, Crollen, Baumans, Lepore, & Collignon, 2016; Kanjlia, Lane, Feigenson, & Bedny, 2016; Kanjlia, Feigenson, & Bedny, 2018, 2021), and even fewer have tested blind children's number knowledge (Cappagli et al., 2022; Crollen, Mahe, Collignon, & Seron, 2011a, 2011b; Crollen, Warusfel, Noël, & Collignon, 2021; Sicilian,

[1] Children were identified as cardinal principle knowers using Wynn's (1990, 1992) Give-a-Number task if they could reliably construct sets of 5 or more objects when asked – e.g., "Can you give me *five* fish?" (see: Schneider et al., 2022)

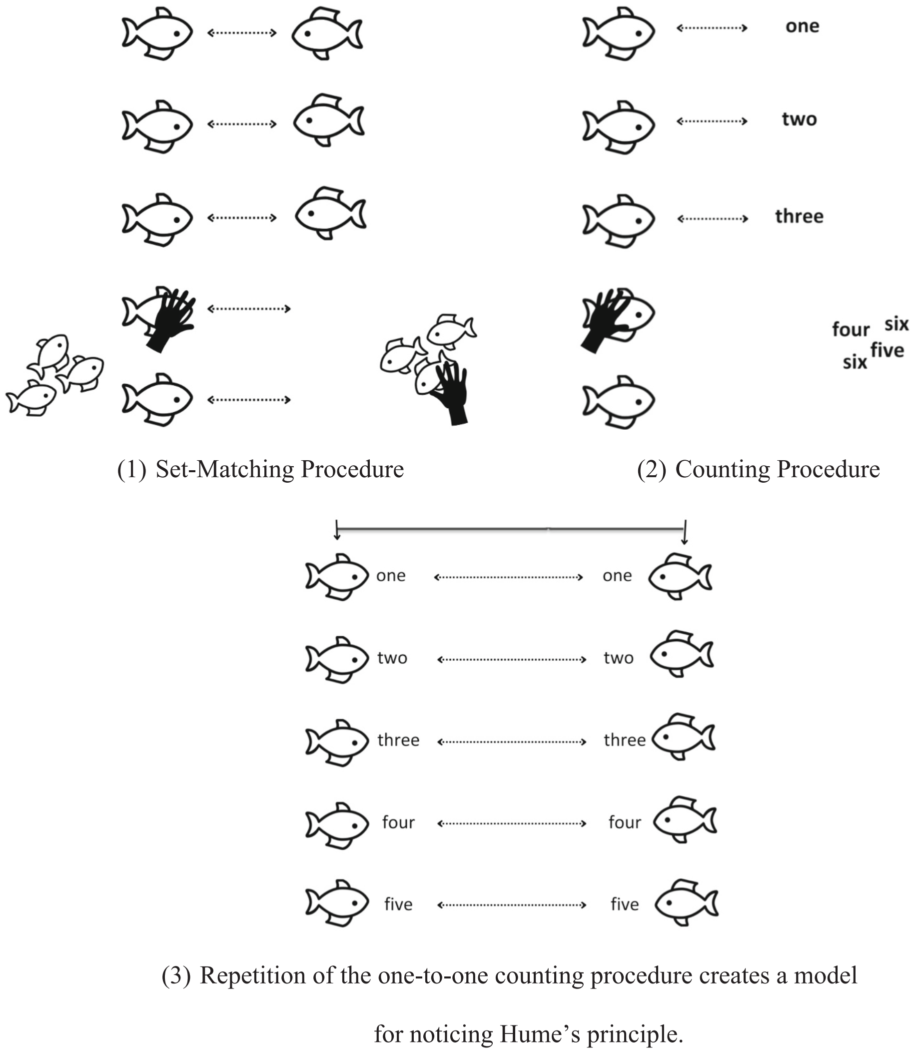

Fig. 2. A graphic depiction of Schneider et al.'s procedural learning hypothesis, under which (1) Hume's principle requires knowledge of symbols, but also requires us to understand a procedure for solving the partition problem (top left); and (2) children learn one such sequential partition procedure when they acquire a counting system (top right). Upon repeatedly using this partitioning procedure to count and construct sets, children notice the same one-to-one relations between the objects themselves, extending the analogy between number words and sets to two sets themselves (bottom).

1988) or other cognitive abilities (Campbell, Casillas, & Bergelson, 2024; Landau, 1986; Landau, Gleitman, & Landau, 2009; Landau, Gleitman, & Spelke, 1981; Landau, Spelke, & Gleitman, 1984; Marlair, Pierret, & Crollen, 2021). Among existing studies on numerical cognition in blind participants, some find comparable approximate representations among blind and sighted individuals (Castronovo & Seron, 2007a, 2007b; Castronovo & Delvenne, 2013; Kanjlia et al., 2018), and others find that blind children use their fingers to count less often and in non-canonical ways relative to sighted counterparts (Crollen et al., 2011a; Crollen et al., 2014). No previous study has investigated blind individuals' understanding of one-to-one correspondence and exact equality, or the strategies they deploy to express such knowledge.

The inclusion of blind individuals provides a compelling test of the role of procedural learning in one-to-one set-matching, by allowing us to probe whether a relation between counting and set-matching persists when the modality of learned procedures is haptic, rather than visual. In a pre-registration of the current study, we reasoned that if sighted children acquire the ability to exactly match sets from an analogy to a sensori-motor counting procedure, then we might expect that blind individuals who have learned a haptic counting procedure should also analogically extend this procedure to haptic set-matching. Consistent with the hypothesis of Schneider et al., months or years of experience deploying a haptic one-to-one counting procedure – a skill which only blind learners must acquire – may provide a template for organizing the difficult task of matching sets haptically, too, by training them to both organize sets in space, and to partition them in a one-to-one procedure. Further, we reasoned that a lack of visual inputs might actually benefit blind individuals when set-matching. In contrast to sighted children who may first rely on quicker, alternative, strategies like visual approximation or matching on the basis of line length, blind individuals cannot apprehend the properties of a target set quickly "in a glance". Instead, any act of perceiving a large set haptically necessarily involves a laborious process of exploring each individual in a set one item at a time. Consequently, the relative advantage of non-exact strategies vs. one-to-one correspondence that exists among sighted participants may not exist in blind individuals. Given this, we might expect blind children and adults to deploy a one-to-one strategy for set-matching under conditions in which sighted individuals do not.

To explore these questions, we investigated set-matching in blind children and adults in sites distributed across India, including the Delhi National Capital Region (in Northern India), Pune (in Western India), and Bengaluru (in the South), representing diverse linguistic, cultural, and educational backgrounds. Unlike in Western, high income countries, where most previous studies of cognition in blind individuals have been conducted, childhood blindness in India is highly prevalent, affecting 1 in 1000 individuals. Also unique to India, these cases often stem from preventable causes without significant comorbidity (e.g., refractive errors, untreated cataracts, glaucomas, and retinopathy of prematurity; see Monga et al., 2009; Vashist et al., 2022). Consequently, unlike in most previous studies of blind individuals which tested small groups of 15–30 participants from mostly western, educated, groups — potentially limiting generalizability of their findings (for discussion, see: Henrich, Heine, & Norenzayan, 2010; Wen et al., 2025; Yazdi, 2022) — we were able to recruit large samples of both children and adults, which allowed us to predict their behaviors using statistical models that included multiple predictors, while also documenting learning in a population that is poorly documented in the literature.

To assess the role of counting knowledge in set-matching ability, we tested each participant on a version of Schneider et al.'s (2022) set-matching task, as well as three measures of counting skill used in previous studies. First, the Highest Count (e.g., Davidson, Eng, & Barner, 2012; Schneider et al., 2020; Schneider et al., 2022) task probed overall familiarity with number words, by asking individuals to count as high as they could. Second, the Give-a-Number task (Wynn, 1990, 1992) was used to evaluate comprehension of number words, and whether children could reliably use counting to label and construct large sets (e.g., greater than 3 or 4). Third, the How Many task (Cantlon, Fink, Safford, & Brannon, 2007; Schneider et al., 2022) was used to assess whether, when counting large sets, participants used a one-to-one counting procedure to assign one number word to each object in a set.

1. Study 1

In Study 1, we tested blind adults and children to ask two questions: (1) do numerate blind participants use a one-to-one correspondence strategy to exactly match sets when prevented from counting? (2) what role does prior counting knowledge (i.e., knowledge of the cardinal principle) and counting proficiency (i.e., as measured by the "How Many?" task) play, if any, in children's set-matching? In doing so, we examined whether mastery of haptic one-to-one counting procedures transfer to the problem of one-to-one set-matching.

1.1. Method

1.1.1. Participants

We tested 50 blind adults ($M = 24.46$; $SD = 5.72$; range = 18–40 years; 26 female) and 50 blind children ($M = 9.04$; $SD = 2.17$; range = 4;11–13;10 years; 10 female) recruited from seven schools and training institutes for the blind across two cities in India: Delhi NCR and Bengaluru, Karnataka. In India, visual impairment is classified into four categories based on severity. Those who received a 100 % classification (i.e., visual acuity <3/60 in better eye; Supplementary Table 1) were included in the study, consistent with ICD-10 criteria. We followed this criterion because in most cases the precise onset and cause of blindness were not known because individuals did not receive reliable medical diagnoses. This is explained, in part, by the fact that most children and adults travel to attend residential schools and rehabilitation programs from remote villages and towns in rural India, where access to medical facilities and general awareness of visual disabilities is severely limited. Although all but 9 adult participants provided information, to encourage and maximize participation from all blind individuals, we did not require them to disclose personal information about medical histories and etiologies as a prerequisite for inclusion in research. Among adults who volunteered this information, 34/50 (68 %) reported congenital blindness, 1/50 (2 %) reported vision loss in infancy (i.e., within the first year of life), 6/50 (12 %) early in childhood (i.e., between 1 and 5 years of age), and 9/50 (18 %) did not disclose this information. Among children, we relied on teacher reports, which indicated that 47/50 (94 %) children were congenitally blind, 1/50 (2 %) lost their vision around 1 year, and 2/50 (4 %) children lost their vision sometime between 6 months and 5 years of age.

1.1.1.1. Education. Adults were recruited from short-term training programs designed to facilitate rehabilitation, and to generate opportunities for gainful employment for blind and low vision individuals. At these programs, adults are trained in mobility and orientation, reading and writing braille, using voice-to-text phone apps, typing on computers, as well as vocational skills such as sewing, candle-making, paper crafts, hand weaving, haptic medical examination, and massage therapy, though specific training opportunities available to participants vary by program. Prior to entering these programs, adults ranged from having no formal education ($n = 5$) to just a few years [up to grade 5 ($n = 2$), grade 6 (n = 2), grade 7 (n = 2), grade 8 (n = 5), grade 9 ($n = 2$)], to those who had completed their secondary school certificates (i.e., grade 10; $n = 10$) or senior secondary certificates (i.e., their high school diploma; $n = 8$), and those who were pursuing (n = 2) or had completed college degrees ($n = 5$). We recruited children from formal schools for blind and low vision children, where they completed the same curriculum as sighted, neurotypical children enrolled in public schools. Although schools typically begin formal schooling at the same time as schools for sighted children, the age of enrolment at blind schools is highly variable, with blind and low vision children beginning schooling anywhere between 4 and 17 years of age, often due to limited awareness. In our sample, onset of formal education was at age 4 ($n = 6$), age 5 ($n = 5$), age 6 ($n = 12$), age 7 ($n = 4$), age 8 ($n = 9$), age 9 (n = 9), age 10 ($n = 3$), and age 12 ($n = 2$). Given these variable ages of enrolment, children ranged from no having formal schooling (n = 9) to 1 year ($n = 16$), 2 years ($n = 18$), 3 years (n = 3), 4 years (n = 2), and 5 years (n = 2) of formal schooling (up to class 5) at the time of test.

1.1.1.2. Language and culture. Participants in both studies were culturally diverse, representing multiple linguistic, religious, and ethnic groups. Although all participants spoke either Hindi or English (and were tested in one of these languages by the same bilingual experimenter), most also spoke additional languages and regional dialects, such as Kannada, Marathi, Bengali, Garhwahli, Punjabi, Tamil, and Telugu, reflecting the linguistic diversity of the country. Language proficiency for each participant was determined through teacher report and medium of counting instruction at the school. Residents of Delhi (population 24 million) speak 94 native languages and dialects, with most speaking Hindi (78.13 %), followed by Punjabi (3.22 %), Bengali (2.33 %), Tamil (1.94 %), and Malayalam (1.92 %), while residents of Bengaluru (population 12.5 million) speak 182 native languages and dialects, including Kannada (44.45 %), Tamil (15.20 %), Telugu (13.99 %), Urdu (12.11 %), and Hindi (4.55 %). Although we did not ask individuals about their cultural backgrounds, a majority of residents of Delhi practice Hinduism (80.21 %), while other prominent religious traditions include Islam (12.78 %), Sikhism (4.43 %), Jainism (1.39 %), Christianity (0.96 %), Buddhism (0.12 %). In Bengaluru, which is located in the Mysore Plateau in the South of India, a majority of people practice Hinduism (78.87 %), followed by Islam (13.90 %), Christianity (5.61 %), Jainism (0.97 %), Sikhism (0.15 %), and Buddhism (0.06 %).

1.1.1.3. Socio-economic background. Participants in our studies came from families with modest means. A majority of adults in our sample came from one-income households, where the primary breadwinners (typically a parent, sibling, or spouse) worked as laborers and masons, chauffeurs, shopkeepers, tailors and seamsters, and farmers, with salaries ranging between US $417.11 and US $4171.14 annually ($M =$ US $1643.72). Although we did not collect these data from children, school and center administrators reported that, similar to adults in the study, the family members of children were typically employed as farmers, rickshaw pullers, chauffeurs, housekeepers, shopkeepers, vegetable vendors, security guards, manual laborers, etc. In these jobs, employees usually receive wages at or below minimum wage, which range in India from $5.82 to $7.64 per day, or roughly $2118.48 to $2780.96 annually, assuming a seven day work week. Consistent with this, when this study was conducted in 2024 the per capita income across India was US$2878 (Government of India, 2025; International Monetary Fund, 2025), and the average annual income across major metropolitan cities in India, including Delhi and Bangalore, as well as Pune (where participants were tested in Study 2) was US $4603 (HomeCredit, 2024).

1.2. Materials and procedure

All tasks were completed in the following fixed order: (1) Set-Matching, (2) How Many? (3) Give-a-Number, and (4) Highest Count.

1.2.1. Set-matching

As in previous work (Gordon, 2004; Schneider et al., 2022) participants were shown a set of objects and asked to match it with another set, and were prevented from counting. The task was conducted in three phases: (1) familiarization, (2) training, and (3) test.

Familiarization. Participants were told that the goal of the game was to make their "pond" (4.5"x30" rectangle) look like the experimenter's pond. At the beginning of the task, the experimenter presented participants with two empty ponds placed in parallel to each other and told them to haptically explore the two blank pieces of cardboard. Participants were encouraged to examine the full extent of the two boards. Next they were introduced to a box containing 15 identical plastic fish placed within the participant's reach, and told that all fish in the box were the same. Participants were given time to explore the two empty boards and box of fish. The experimenter began the task by saying, "Ready? This is a matching game. Do you know what matching means? Matching means to make things look the same! So, in this game, you're going to make your pond look like my pond."

Training. The experimenter then placed a board with one fish glued to its center directly above the child's pond, and said, "I'm putting a fish here in my pond. Without counting, can you make your pond have the exact same number of fish as my pond?" Participants were trained on sets of 1 and 2 fish. On the trial with 2 fish (and all following test trials), fish were placed at 2.5 cm distances from each other. If participants responded correctly on the first training trial, the experimenter said, "That's right! Your pond looks like my pond because there is a fish here (the experimenter took the participant's hand to indicate the fish on the experimenter's board) and a fish here (the experimenter took the participant's hand to indicate the fish on the participant's board). Great job! You made the ponds match." However, if the participant did not respond correctly, the experimenter provided corrective feedback "Hmm, I don't think these ponds match," and took their hand to identify the discrepancies", in one of two ways. First, if the participant placed more fish than necessary, the experimenter said "There is one fish here (the experimenter took the participant's hand to indicate the fish on experimenter's board), but there is more than one fish here (the experimenter took the participant's hand to touch all the fish on participant's board). Let's try again." The experimenter then emptied the participant's board and repeated the instructions. Second, if there were fewer fish than requested on the board (i.e., if the participant only placed 1fish in a 2 fish trial), the experimenter said: "There is one fish here (the experimenter took the participant's hand to indicate the fish on the experimenter's board) and one fish here (indicating the fish on the participant's board), and there is one fish here (second fish on experimenter's board), but no fish here (identifying the empty spot on the participant's board). Can you make it so that your board has the same number of fish as my board?" Once the participant corrected their errors, the experimenter provided positive feedback, repeating the identification procedure for correct trials. Participants were given up to 2 trials for each training set (i.e., a maximum of 4 training trials were administered), and their final response was recorded. In doing so, participants were given both an opportunity to explore the empty spatial layout prior to sets being placed and were taught the one-to-one relations between the objects comprising the two sets. Only those who correctly matched for these trials were included in the study to ensure that all participants understood the task. Only 6 children and 1 adult across the 2 studies were excluded for not passing training.

Test. After training, participants were tested on 2 small sets (3, 4) and 3 large sets (6, 8, 10). Instructions were repeated at the beginning of each test trial. Trial order was fixed and counterbalanced across participants: (3, 4, 10, 8, & 6) or (4, 3, 8, 10, & 6). No feedback was provided in between trials (Fig. 3).

1.2.2. How Many?

This task tested participants' ability to assign each object in an array a unique number word, analogous to the one-to-one procedure that they must use to exactly match sets of objects (Cantlon et al., 2007). Participants were asked to count two large sets (8, 10), with the order of trials randomized between subjects. Following Schneider et al.'s (2020) coding scheme, participants received a "counting proficiency" score between 0 and 3 for each instance trial: 3 = errorless counting (one-to-one coordination of words and objects); 2 = correct count with errors fixed (e.g., by restarting); 1 = correct counting for a at least 2 items; 0 = random counting.

1.2.3. Give-a-Number

This task (Wynn, 1990, 1992) was used to test participants' ability to use counting to generate sets. Following Schneider et al. (2020), we classified participants into Cardinal Principle knowers (i.e., who could use the counting procedure to generate large sets) and subset knowers (i.e., who could not use the counting procedure to generate large sets). Participants were shown a bowl of plastic fish and asked to create sets of N fish, starting with 5. If they failed to give N, the experimenter asked them to count to verify if they gave N. If they correctly gave N, the next number queried was N + 1 (up to 6). If they were incorrect for N, the next number was N-1. This staircase procedure established the largest number correctly given on at least 2/3 trials. Participants who gave correct responses up to 6 were coded as Cardinal Principle knowers (i.e., accurate counters). The others were coded as Subset Knowers (those who know a subset of numbers).

1.2.4. Highest count

Participants were asked to count as high as they could as a proxy for exposure to counting (e.g.,Davidson et al., 2012; Schneider et al., 2022). If they stopped at a decade label (e.g., twenty), they were prompted to "keep going", and "count as high as you can." The task was stopped if the participant made an error, or counted to 120 without error. The number reached before stopping or making an error was recorded as the highest count.

1.3. Results

To assess participants' ability to use one-to-one correspondence to

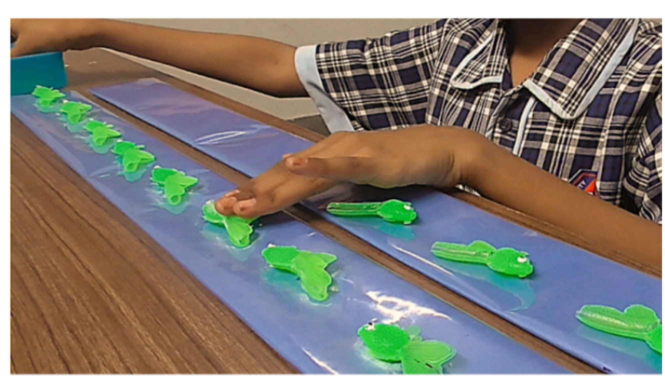

Fig. 3. Image of a blind child using a one-to-one strategy to match a large set.

match large sets in each experiment, we constructed pre-registered logistic mixed effects models predicting accuracy on the set-matching task. In both experiments, we began with a base model, and systematically added or removed terms to determine which variables explained unique or combined variance in participants' accuracy. Best fitting models were chosen based on a lower AIC value and significant chi-squared statistic. All post hoc pairwise comparisons were adjusted for multiple comparisons using Bonferroni correction. Participant age was a fixed term in all models.

We characterized numeracy in three ways. First, we asked if participants understood that the last word in a count represents the cardinality of the counted set, using the Give-a-Number task (Wynn, 1990, 1992). All participants except one child used counting to generate large sets and were therefore classified as "Cardinal Principle Knowers".

Second, we asked if participants could use a one-to-one counting procedure, matching each number word to a unique object, in the "How Many" task (Cantlon et al., 2007). Participants counted 2 large sets (8, 10) and were assigned a score from 0 to 3 based on accuracy (see Methods). Participants demonstrated high proficiency in this task [children: $M = 2.76/3$ ($SD = 0.58$); adults: $M = 2.92/3$ ($SD = 0.27$)], and most performed perfectly on the task (children 42/50; adults 46/50), suggesting they could apply a one-to-one procedure to count sets, similar to sighted individuals (Fig. 4).

Finally, participants were asked to count as high as they could, and we recorded their Highest Count before they stopped, made an error, or reached 120 (Davidson et al., 2012; Schneider et al., 2022). Both children and adults showed mastery of the verbal count list and counted to at least 100 on average [children: $M = 100.32$ ($SD = 34.92$); adults: $M = 115.88$ ($SD = 34.92$)], and a majority had a highest count over 100 (children 39/50; adults 46/50; Fig. 5). Thus, participants could construct exact numbers by counting, use a one-to-one counting procedure, and recite numbers proficiently.

To test our primary questions, we constructed a preregistered base model [Accuracy ~ Set Size (small / large) + Age + (1|PID)] and then added measures of counting (i.e., How Many & Highest Count). We first report data from adults, and then children. Similar to past findings in innumerate groups and semi-numerate sighted children, we found that blind adults were much less likely to exactly match large sets relative to small ones ($B = -2.98, p < 0.001$; Fig. 6), suggesting that they were not using a one-to-one correspondence procedure to complete the task. Age was not a significant predictor of set-matching accuracy in this base model ($B = 0.07$, *n.s.*). Interestingly, performance on neither the Highest Count ($B = 0.10$, *n.s.*), nor the How Many task ($B = 0.13$, *n.s.*) predicted exact matching. Thus, individual differences in numeracy did not predict performance on this task, likely because adults were near ceiling in

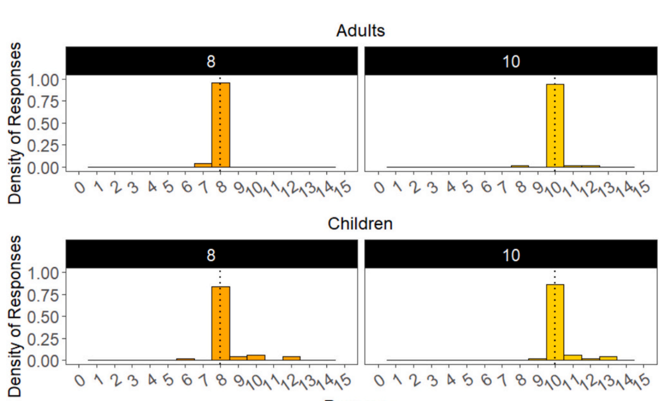

Fig. 4. Distribution of final responses on both trials of the How Many? task in adults (top) and children (bottom). The y-axis depicts density of responses and the x-axis represents the final count produced, faceted by target set size (8,10). The dotted line represents the target set.

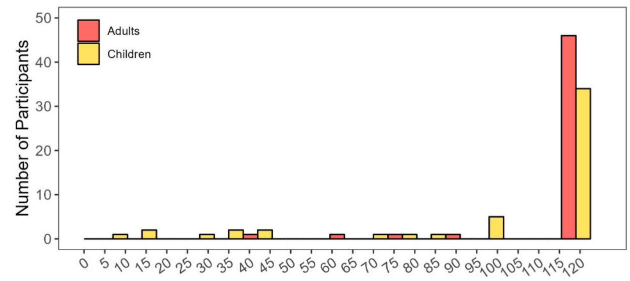

Fig. 5. Distribution of highest counts reached by adults (red) and children (yellow) next to each other. The y-axis represents the number of participants and the x-axis represents their highest count. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

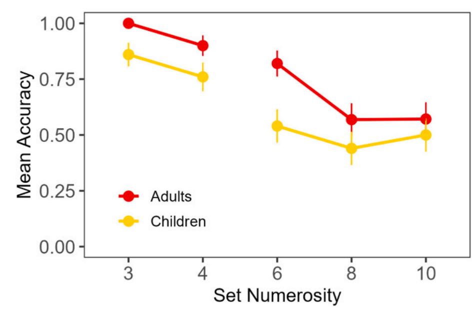

Fig. 6. Mean set-matching accuracy for Experiment 1, divided by adults and children. Error bars represent mean standard error.

numeracy measures. Like adults, children were less likely to exactly match large sets versus small ones ($B = -3.66, p < 0.001$), though accuracy improved with age ($B = 1.49$, $p = 0.04$). Children's Highest Count ($B = 0.03, p < 0.05$) was a significant predictor of their ability to make exact matches, and adding a term for Highest Count significantly improved model fit ($\chi^2(1) = 9.72, p = 0.001$). However, their How Many score did not predict set-matching accuracy ($B = 1.51$, *n.s.*), and adding a term for it did not improve model fit ($\chi^2(1) = 1.52$, *n.s.*). In a follow-up experiment (Supplementary Materials) with a new group of 24 children, we replicated the finding that children were not using one-to-one correspondence when matching large sets, even when provided with extra scaffolding, objects with simpler shapes, and more spacing between items. These changes made no difference, and results did not differ.

Early in the testing process we noticed that some participants appeared to use visual cues to pair objects (by bringing objects to their eyes, and closely inspecting them as they were being paired). Based on the experimenter's subjective judgment, around a third of adults ($n = 14$; 28 %) and children ($n = 18$; 36 %) used some visual information to exactly match sets. Others did not appear to have such visual sensitivity, and relied solely on haptic exploration to complete the task. In post hoc analyses, we found that both adults ($B = 2.02, p = 0.01$) and children ($B = 4.59, p < 0.001$) who appeared to rely on visual cues were more likely to make exact matches than those who appeared to rely solely on haptic exploration (Fig. 7a, 7b, and 7c). Compatible with this, adding a term for perceptual strategy (visual / haptic) significantly improved model fit in models with both adults ($\chi^2(1) = 7.87$, $p = 0.005$) and children ($\chi^2(1) = 21.08$, $p < 0.001$). In addition, children's age was not a significant predictor of their accuracy on the set-matching task after accounting for perceptual strategy ($B = 1.03$, *n.s.*), though their Highest Count continued to predict accuracy ($B = 1.27, p < 0.006$). A similar pattern of results was found when analyses were restricted to individuals with congenital blindness (we removed 16 adults and 3 children with prior history of visual experience in early infancy or early childhood; see Supplementary Materials). Thus, it appears that individuals who

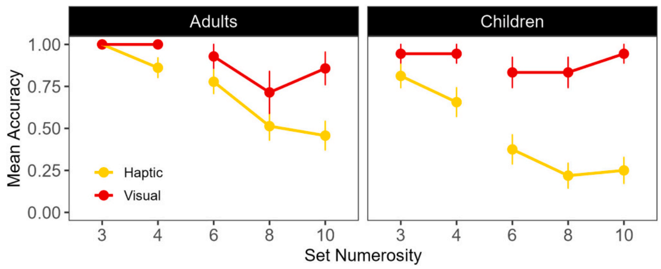

Fig. 7a. Mean set-matching accuracy for Experiment 1, for participants who used visual or haptic strategies. Error bars represent mean standard error.

experienced a gradient of visual inputs were more likely to exactly match large sets relative to participants who had no such visual experience. We explore this result in Study 2.

Lastly, because adults and children in our sample had variable educational experiences, we examined the role of education in individuals' ability to make exact matches in a final set of exploratory analyses. To do so, we added a fixed term for years of education to the best fitting models for both adults and children. Among adults, we found that no significant effect of years of education on set-matching accuracy ($B = 0.04$, *n.s.*), and adding a term for education did not improve model fit ($\chi^2(1) = 0.33$, *n.s.*). Similarly, among children, we found that years of education was not a significant predictor of performance on set-matching ($B = 0.15$, *n.s.*), and a term for education did not improve model fit ($\chi^2(1) = 0.14$, *n.s.*).[2] Finally, because the age at which children began formal schooling was highly variable, we also examined whether the age children enrolled in school – rather than the number of years they had spent in school – predicted their set-matching accuracy. We found no significant effect of age of education onset ($B = -0.31$, *n.s.*), and adding the term did not improve model fit ($\chi^2(1) = 0.68$, *n.s.*). These results suggest that formal education did not contribute to adults' or children's understanding of exact equality.

1.4. Discussion

Study 1 found that blind children and adults often made exact matches for small sets, but were unlikely to make exact matches for larger sets. This was despite the fact that most children and adults in our study were at or near ceiling on our three measures of numeracy, including the Give-a-Number and How Many tasks, which required participants to deploy a haptic procedure for establishing one-to-one correspondence between number words and items in sets. Thus, many individuals who were able to haptically organize sets of objects in space, partition them into counted and uncounted sets, and assign labels to items in one-to-one correspondence were nevertheless unable to use an analogous haptic procedure for set-matching. Finally, in an exploratory analysis, we found that blind participants with a gradient of visual experience were significantly more likely to make exact matches, relative to participants without such experience, providing a compelling within-group test of the role of visual experience in set-matching.

In the Introduction, we raised two questions that are addressed by these data. First, we asked whether the use of a haptic modality, which requires manual exploration of each individual object, might promote the use of one-to-one correspondence since participants cannot easily apprehend the magnitude of a set in a single perceptual "glance". Against this, we found that blind participants seldom made exact matches, and that actually the likelihood of exact matching was greater among participants who appeared to use visual cues during the task. Second, we tested the procedural analogy hypothesis of Schneider et al. (2022), and found that although our participants had acquired a haptic procedure for organizing sets in space and creating one-to-one correspondence between words and objects while counting, they did not analogically extend this procedure to the problem of set-matching. Instead, participants who had exact number knowledge — and could accurately count, construct, and label sets larger than 3–4 — rarely made exact matches for large sets, indicating an important dissociation between symbolic exact number knowledge and set-matching ability.

These data provide strong evidence that blind individuals do not make a procedural analogy between counting and set-matching. Consequently, our data suggest that either such an analogy plays no role in the origin of exact matching abilities – whether in sighted individuals or blind learners – or alternatively that a procedural analogy between counting and set-matching is more available visually than haptically.

Crucially, although our data suggest that haptic counting experience do not support the creation of an effective haptic set-matching procedure, they are nevertheless compatible with the hypothesis that when blind participants make non-exact matches, this is because they lack an effective procedure for establishing one-to-one correspondence, and not because they lack conceptual understanding of Hume's principle. In particular, this possibility is raised by the dissociation we found between participants who appeared to use visual cues for set-matching versus those who did not.

Following the logic of studies like Gordon (2004), one possibility is that blind participants in our study lacked conceptual understanding of Hume's principle, unlike individuals who have a gradient of visual experience, or are fully sighted. A history of visual experience may be crucial to developing a complete understanding of one-to-one correspondence. Another possibility, however, is that all participants in our study understood Hume's principle and what differentiated them was their access to an effective sensori-motor procedure for organizing objects in space, and for creating a one-to-one correspondence between items within each set. Relevant to this, learning an effective procedure in the haptic modality may be more difficult because of the unique challenges of organizing space haptically. For example, previous studies of geometry, navigation, and tactile memory all suggest that spatial processing in the haptic modality is more challenging than via visual cues (Heimler et al., 2021; Marlair et al., 2021), and that blind individuals process spatial relationships differently from sighted peers (Crollen et al., 2021; Landau et al., 1984). Consequently, the problem of constructing an effective haptic procedure for set-matching may be more difficult, even when beginning with an analogous template learned to haptically count sets of objects. On this second hypothesis, rather than supplying conceptual understanding, access to visual cues may provide a more effective and easy-to-implement procedure relative to haptic information.

In Study 2, we explored these ideas further by investigating blind individuals who have a gradient of visual experience, and by testing them twice: once with access to visual information, and once blindfolded. On the hypothesis that conceptual knowledge differentiates participants in Study 1, but that procedural learning is not a barrier, we should expect individuals who have a gradient of visual experience to make exact matches even when blindfolded during the set-matching task. However, if the key difference between participants in Study 1 was not conceptual, but instead explained by the greater difficulty of deploying a haptic procedure, then we should expect participants to make exact matches only when they are able to actively use visual cues as part of a matching procedure, but not when blindfolded.

Note that a critical limitation of Study 1 was that, to code the use of visual cues, the experimenter observed the set-matching behaviors of participants, raising the possibility that coding was biased by how

[2] Note that adults' education qualifications may not translate directly to years of education, since some may have obtained certifications via equivalence tests. For consistency across adult and child analyses, adult qualifications were transformed to the equivalent number of years typically required to obtain them. For example, a senior secondary certificate (equivalent to a GED) takes at least 12 years in India. Similarly, a college degree is 3 years, representing 15 years of education.

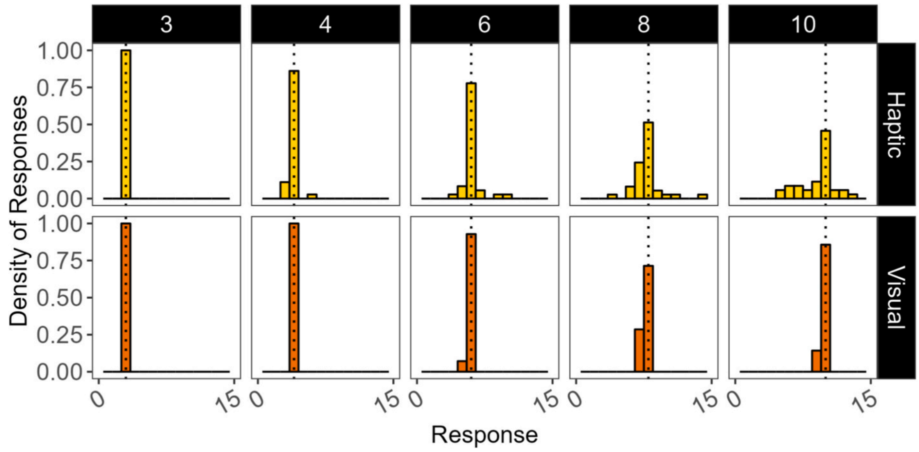

Fig. 7b. Distribution of adults' set-matching responses for sets of 3, 4, 6, 8, and 10 in Study 1, grouped by strategy (i.e., visual or haptic). The dotted line represents the target set.

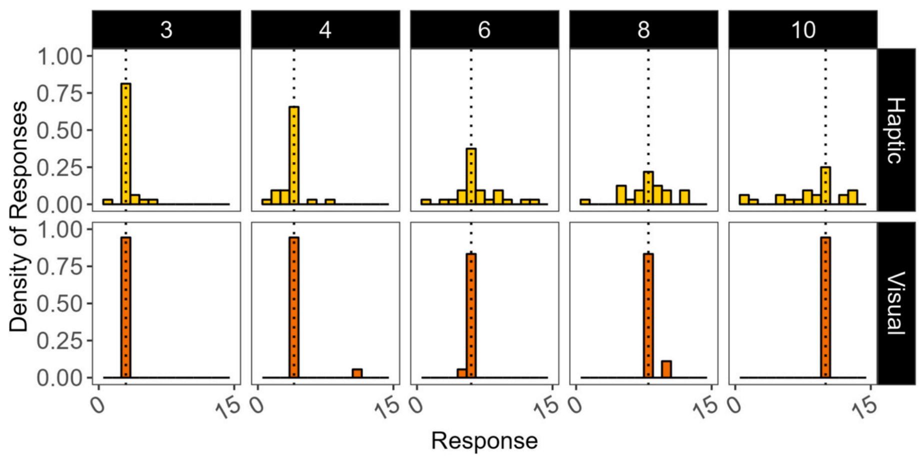

Fig. 7c. Distribution of children's set-matching responses for sets of 3, 4, 6, 8, and 10 in Study 1, grouped by strategy (i.e., visual or haptic). The dotted line represents the target set.

participants performed on the task. Given this, in Study 2 we tested a new group of participants, all of whom showed evidence of using visual cues in a pre-test that did not involve set-matching. Each individual was tested twice: once with a blindfold to obscure visual cues, and once without. We reasoned that if the active use of visual cues impacts set-matching, then blindfolding should reduce exact matching.

2. Study 2

In Study 2, we asked what role procedural information plays among blind individuals who have a gradient of visual experience. One possibility is that a history of visual experience is sufficient to furnish an understanding of exact number and one-to-one correspondence, and that children who are able to use visual information during the set-matching task are also able to succeed when using haptic strategies alone (e.g., when the sets are occluded and they cannot use visual information). For blind children with a history of using haptic information to explore and move around the world, the use of a haptic procedure may not pose any additional challenge. Another possibility, however, is that a haptic set-matching procedure is especially difficult to implement, even for blind individuals who use haptic information to navigate the world on a daily basis, and that children who succeed when permitted to use visual cues are unable to exactly match large sets when they are blindfolded and required to use haptic cues alone. To evaluate these possibilities, we tested each child twice: once blindfolded, to remove visual access, and once without a blindfold. If the ability to exactly match large sets depends upon visual access to the sets during the task, then we expect blindfolding to reduce performance among children who appear to rely on visual cues, but not among those who do not.

2.1. Method

2.1.1. Participants

Because children and adults who appeared to use visual information in Study 1 did not differ from each other in set-matching accuracy ($B = -0.10$, *n.s.*; Fig. 7a), we only tested children in Study 2. This study included 40 blind children ($M = 8.07$; $SD = 1.77$; range = 4;7–12;5 years; 12 female). As in Study 1, children with a visual acuity of <3/60 in the better eye were included in the study. Based on teacher reports, at least 39/40 (97.5 %) children were congenitally blind, and 1/40 (2.5 %) children experienced vision loss at 5 years of age.

In addition to recruiting children from Delhi (see Study 1), children in Study 2 were also recruited from Pune (population 4.5 million), a city situated in the Deccan plateau in Western India. Residents of Pune speak 177 native languages and dialects, including Marathi (78.07 %), Hindi (10 %), Urdu (1.8 %), Kannada (1.40 %), and Marwari (1.34 %). A majority of residents practice Hinduism (79.43 %), followed by Islam (11.03 %), Buddhism (3.94 %), Jainism (2.45 %), Christianity (2.17 %), and Sikhism (0.43 %). Data related to household income are also reported in Study 1.

2.2. Materials and procedure

All tasks were administered in the following fixed order: (1) Counting Prescreen, (2) Set-Matching (with visual access or blindfolded); (3) Give-a-Number, and (4) Highest Count.

2.2.1. Counting prescreen

As an inclusion requirement, children were screened for whether they used visual cues to count sets, or only used haptic cues. They were asked to count sets of 8 or 10 fish and the experimenter coded if they used visual cues, including if (1) they did not use touch to explore the objects, (2) they appeared to move their head to scan sets (e.g., bringing it close to the objects, tilting their head to one side to use peripheral vision), or (3) they brought objects close to their face. Those who ostensibly used vision were recruited for the study.[3]

2.2.2. Set-matching

The set-matching task was administered to each participant twice, once allowing visual access, and once blindfolded, counterbalanced between participants. Trial order (3, 4, 10, 8, & 6) or (4, 3, 8, 10, & 6) was counterbalanced.

2.2.3. Give-a-number & highest count

Because we found no effect of the How Many? task on set-matching performance in Study 1, this measure was eliminated in Study 2. The Give-a-Number and Highest Count tasks were administered in the same way as Study 1.

2.3. Results

Based on the prescreening task, only children who showed evidence of using visual cues were included in the study. We asked whether children who could make exact matches when given visual inputs – and thus showed an understanding of Hume's principle – would also make exact matches when blindfolded, or if they would be unable to express this knowledge when blindfolded, because they lack an effective non-visual procedure for establishing one-to-one correspondence. Children again exhibited high Highest Counts ($M = 74.46$; $SD = 41.7$), and all but 4 ($n = 36$; 90 %) were Cardinal Principle knowers.

As in Study 1, we preregistered a base model [(Accuracy ~ Set Size (small / large) * Visual Access (present / absent) + Age + (1|PID)], and additional terms were added to examine the role of the order in which children completed the task (i.e., with visual access first, or with blindfolds first), and the role of Highest Count in predicting set-matching. Children were significantly more likely to exactly match sets when they had access to visual information than when blindfolded ($B = 1.28$, $p = 0.01$), and overall they were more likely to make exact matches for small sets than for large sets, regardless of whether they had visual access ($B = 1.34$, $p = 0.01$) or were blindfolded ($B = 1.24$, $p = 0.005$; Fig. 8). Once again, age did not predict set-matching accuracy ($B = -0.08$ *n.s.*), even when only non-blindfolded trials were considered ($B = -0.28$ *n.s.*). Similarly, Highest Count did not predict accuracy on set-matching overall ($B = -0.45$, *n.s.*), nor when only non-blindfolded trials were considered ($B = -0.86$ *n.s.*), and adding a term for Highest Count did not improve model fit ($\chi^2(1) = 2.07$, *n.s.*).

Interestingly, children who completed the visual version of the task first were more likely to match large sets on the visual version relative to the blindfolded version ($B = 0.15$, $p < 0.001$). However, those who were blindfolded first did not show a similar difference between the two tasks ($B = 0.53$, *n.s.*). This suggests that whereas the use of a visual one-to-one strategy did not transfer to the use of one-to-one haptically, a haptic approximation strategy in block 1 may have interfered with performance on trials in which visual cues were available.

2.4. Discussion

Study 2 replicated the primary findings of Study 1, but also found that when blind children with limited visual inputs were blindfolded, their exact matches declined relative to when they were not blindfolded, resembling the performance of individuals who completely lacked visual inputs. We also found that children who completed the visual version of the task first were more accurate on the visual version relative to the blindfolded version, suggesting that performance declined in the haptic modality despite having visual experience of the sets. Moreover, children who were blindfolded first were less likely to use a one-to-one strategy when tested on the visual task, suggesting that if children did not use one-to-one correspondence in the haptic modality, they were less likely to transition to using a one-to-one strategy even when visual cues were made available to them. Together, these results suggest that difficulties implementing one-to-one correspondence were not conceptual, but reflected the problem of executing an effective haptic procedure to produce exact matches. Individuals who demonstrated a concept of exact number and who could use haptic one-to-one procedures for organizing and counting sets did not analogically extend this procedure to the problem of set-matching.

3. General discussion

In two studies, we investigated how numerate blind children and adults match sets, and found three main results. First, in Study 1, we found that despite having symbols to express large exact numbers, blind children and adults generally did not use one-to-one correspondence to exactly match large sets despite easily matching smaller sets. However, in an exploratory analysis that examined within-group effects of visual perception, we also found that participants who appeared to use visual cues were more likely to make exact matches. In Study 2, we conducted a within-subjects test of the role of visual experience, and asked whether blind children with a gradient of visual input could also make exact matches haptically, when blindfolded. We found that they could not, suggesting that although these children understood Hume's principle and could match large sets exactly in the visual modality, they failed to match sets when this required the deployment of a haptic modality, despite the availability of an analogous one-to-one procedure used for counting.

These results support three main conclusions. First, they suggest that blind individuals who use a haptic procedure for organizing and counting sets do not extend this procedure to the problem of matching sets, contrary to the predictions of the procedural analogy hypothesis of Schneider et al. (2022). Second, the fact that blind individuals with a gradient of visual experience are less likely to make exact matches when blindfolded suggests that their challenge is modality specific, and not conceptual, and that in particular they lack an effective haptic procedure for organizing sets and placing them into one-to-one correspondence, despite having an analogous counting procedure. Thus, although we do not find evidence for the procedural analogy hypothesis, we nevertheless find support for the idea that set-matching requires both conceptual understanding of Hume's principle and modality-specific procedural knowledge. Finally, the results suggest the broader conclusion that when individuals fail to make exact one-to-one matches, this may not necessarily reflect a conceptual deficit, but may instead indicate a lack of experience organizing sets in space, partitioning them, and placing them into one-to-one correspondence. Such procedural skills, whether they arise from counting experience or elsewhere, may in part explain cross-

[3] Note that only one experimenter was available to code behaviors, and we did not have permission to record most sessions. We were therefore unable to conduct reliability coding for participant classification. Crucially, any misclassification of a participant as having a gradient of vision should reduce the likelihood of detecting an effect of blindfolding (since a misclassified individual would lack visual inputs in both conditions). Thus, any such misclassifications could not easily explain the pattern of results that we report.

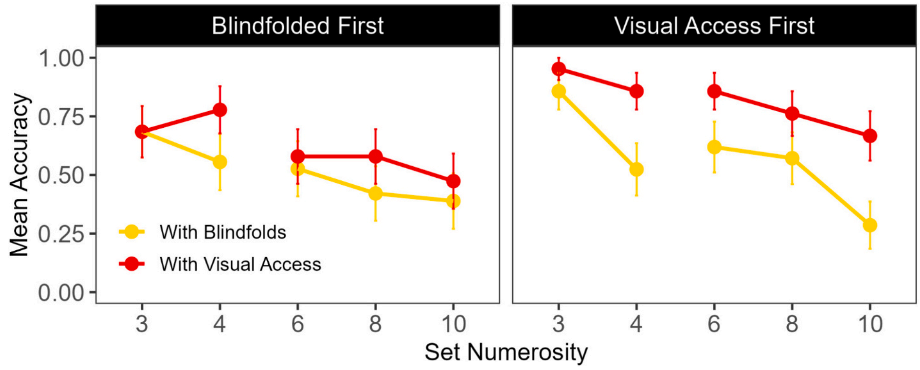

Fig. 8. Mean set-matching accuracy for Experiment. 2, for Visual Access & Blindfold conditions (right). Error bars represent mean standard error.

cultural differences in set-matching behaviors.

These three conclusions suggest a reassessment of how the acquisition of symbolic number – like number words and counting procedures – might impact a global understanding of exact number. The results reopen the debate regarding whether innumerate groups and young children truly lack understanding of Hume's principle, or simply lack procedural knowledge. But even on the view that only procedural knowledge is missing, there remain many possible ways in which numeracy and set-matching might be related. One possibility is that set-matching is entirely independent of skills like counting, and acquired via other cultural practices that have yet to be measured, and that are present among numerate, sighted, individuals but not in innumerate groups, blind individuals, or young sighted children. Another possibility, however, is that mastery of counting procedures does play some role, but that the information that it provides is accessible only in the visual modality, and cannot be as easily deployed haptically. For example, it's possible that Schneider et al. (2022) are right that set-matching is based on an analogy from the one-to-one structure of counting – explaining why only numerate individuals make exact matches, and why this is correlated with counting ability in sighted children – but that making this analogy depends upon information that is only easily extracted from visual scenes. What's needed, on such an account, is a fine-grained analysis of why blind children can easily partition objects that have been assigned a numerical label from those that have not, while struggling to partition objects that have been assigned another object from remaining unpaired items.

While the present study raises important theoretical questions regarding the general problem of how humans come to express conceptual understanding of exact number, it also sheds important light on how numerical knowledge is expressed in blind individuals, more specifically. The study included relatively large samples of participants from a diverse array of linguistic and cultural groups across rural and urban India, a country in which childhood blindness is highly prevalent and understudied. As such, it constitutes one of the largest studies to assess numerical abilities in blind children and adults. Setting aside the theoretical importance of this work, this approach allowed us to document counting and non-symbolic numerical reasoning in a diverse group of blind participants, and to understand the possible challenges that they face in applying numerical knowledge to concrete spatial layouts. This work points to interesting future directions, aimed at understanding developmental processes among blind learners. For example, an interesting question for future work is whether abilities like haptic set-matching can be easily trained, and exactly what makes this task difficult to execute haptically. Also interesting is whether the training conditions that lead to exact matching of large sets in blind individuals might also improve set-matching in young sighted learners, too, or if qualitatively different factors restrict this ability in these groups.

While conducting this work in India allowed us to include under-represented groups in a context where blindness is highly prevalent and in need of research, this approach posed unique difficulties, and produced several limitations that would ideally be addressed in future research. First, one limitation is that in India, most individuals with untreated blindness live with modest means, and have minimal access to quality medical services. This makes it difficult to obtain detailed information regarding etiology of blindness or other medical or cognitive comorbidities, which might contribute to potential variability in learning. Second, although our sample was likely representative of blind individuals in India, it remains uncertain whether results from these groups generalize well to blind individuals in other countries and cultures. While this is a constraint, most past work on blindness has included relatively small samples of participants from so-called WEIRD societies (Henrich et al., 2010), in which blindness is much less prevalent, and therefore less likely to generalize to a large group of individuals. While one way to address this might be to make direct cross-cultural comparisons, previous studies suggest that a within-group and within-subjects approach, like that taken here, may offer the strongest test of causal claims (Schneider et al., 2020; Yazdi, 2022). For example, by comparing how blind children compare to peers who have a gradient of visual inputs, we manipulated the role of visual access in our experiments, while simultaneously controlling for the many additional factors, aside from blindness, which might otherwise explain differences between sighted and blind individuals, including social and educational experiences, exposure to formal testing environments, familiarity with researchers, parent interactions and home environments, etc. (e.g., Gunderson, Spaepen, Gibson, Goldin-Meadow and Levine, 2015; Hayes & Proulx, 2024; Klingenberg et al., 2019; McDonnall, Cavenaugh, & Giesen, 2012; Spaepen, Gunderson, Gibson, Goldin-Meadow and Levine, 2018; Spinczyk, Maćkowski, Kempa, & Rojewska, 2019). A final note is that in the present study we did not consider how individual differences in domain-general capacities (e.g., spatial working memory), home and school environment, and specific aspects of formal education (e.g., exposure to braille, abacus training, etc.), might impact numeracy. Current studies in our lab, focused on broader measures of numeracy, are beginning to address some of these questions.

In summary, we found that many blind children and adults, who could accurately count and label large sets using haptic procedures, did not exactly match large sets of objects, despite being able to match smaller sets. Also, among those participants who did make exact matches, this appeared to depend on access to limited visual cues. Finally, blind children who appeared to use a gradient of vision to make one-to-one matches – and thus demonstrated an understanding of Hume's principle – did not make exact matches for large sets when blindfolded. These findings suggest that although learning haptic one-to-one procedures for counting may not transfer to the problem of set-matching, procedural knowledge is nevertheless important to expressing an understanding of Hume's principle, and may be especially difficult to master in the haptic modality, or in cultures with limited exposure to the counting procedure. More generally, our findings raise the possibility that the set-matching task may underestimate human understanding of exact number, and may also depend on sophisticated sensori-motor procedures.

CRediT authorship contribution statement

Urvi Maheshwari: Writing – review & editing, Writing – original draft, Visualization, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **David Barner:** Writing – review & editing, Writing – original draft, Supervision, Resources, Project administration, Methodology, Funding acquisition, Conceptualization.

Funding information

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Declaration of competing interest

The authors have no competing interests to declare.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.cognition.2025.106351.

Data availability

https://osf.io/rgudw/overview

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